ANALYSIS OF INTERLOCKING BLOCK PAVEMENTS BY FINITE ELEMENT METHOD

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SUMMARY
For the rational design of interlocking block pavements, it is necessary to make clear the stresses in concrete blocks due to traffic loads. However, there is no theoretical method developed for the analysis of assembled discontinuous structures, such as interlocking block pavements. The Finite Element Method (F.E.M) provides powerful tool to handle discontinuous structures. Then, we developed the computer program for analysis of interlocking block pavements by the use of the F.E.M.
The rectangular plate element was employed to represent the concrete block. The joint is modeled as a set of springs and the stiffness matrix of the joint element is derived by the virtual work principle. The values of the spring coefficients in the joint element should be determined from experiments or field tests. In the computer program, a interlocking block pavement was divided into rectangular plate elments and joint elements and the subbase was assumed to be a liquid foundation or a "Winkler foundation" which is used in the analysis of concrete pavements.
We can compute the stresses and deflections in concrete blocks due to traffic load by this program changing properties and dimensions of blocks, laying pattern, supporting condition and loading condition.

1. INTRODUCTION

Since the interlocking block pavements are consisted of concrete blocks which have high sterngth and excellent durability and easily rehabilitated in the area where blocks are broken and large settlements occur, it can be applied to roads in residential areas as well as to roads in intersection and container terminal where heavy traffic loads are applied.
The mechanical behavior of interlocking block pavements is empirically evaluated by deflections. In Japan, however, there is little tradition and experience of block pavements. Furthermore, because of high strength of concrete blocks and effect of joints between blocks, their mechanical behavior is expected to be very complex. Consequently, for purpose of the rational design of interlocking block pavements for heavy traffic roads, it is necessary to make clear the stresses in concrete blocks. However, there is no theoretical method to evaluate these stresses. The Finite Element Method (F.E.M.) which have been developed along with the development of high speed digital computers provides powerful tool for treating the discontinuous structures, such as interlocking block pavements.
In this paper, the computer simulation model of interlocking block pavements by the use of F.E.M. is presented and its validity is examined by computing the stresses and deflections under the various conditions.

2. METHOD OF ANALYSIS

The interlocking block pavements are consisted of concrete blocks and joints filled with sand; i.e., they are a kind of discontinuous structure. Thus, their mechanical behavior is expected to be very complicated. However, there is no theoretical method available to analyse interlocking block pavements. So far, in order to evaluate the deflections of interlocking block pavements due to loads, the multi-layer-theory is used by replacing the block layer with an equivalent continuous one. This method, however, has a significant disadvantage of being unable to estimate the stresses and strains in concrete blocks due to loads, while the F.E.M. can applied to estimate them by modeling blocks and joints to an assembly of proper finite elements.
In our study, we adopted two dimentional plate-bending F.E.M. which is based on the classical plate bending theory.

2-1. Rectangular plate element

The rectangular plate element employed in our study is shown in Figure 1 (1).

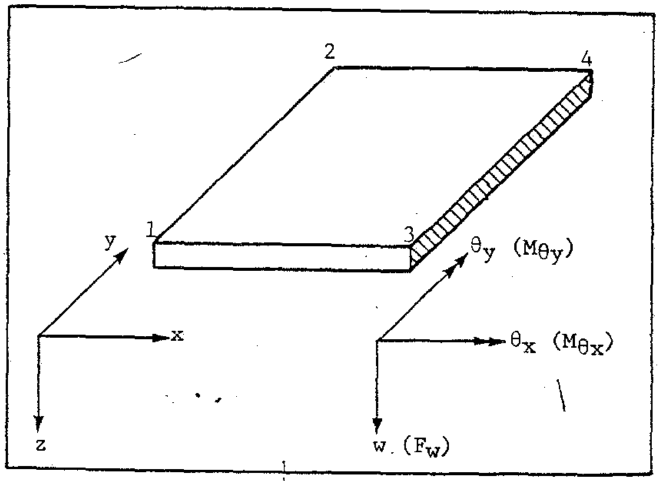


Figure 1: Rectangular plate element.

It has four nodal points, and each nodal point has three fictitious forces and corresponding displacements. The three forces are a vertical force, F_w a moment around x-axis, M_x and a moment around y-axis, M_y. The three displacements are a deflection in the z-direction, w; a rotation around x-axis, θ_x and a rotation around y-axis, θ_y. The three forces and displacements have the following relation:

$$\mathbf{f} = (\mathbf{K} + \mathbf{H}) \cdot \mathbf{d} \tag{1}$$

in which

$\mathbf{f} = < f_1\ f_2\ f_3\ f_4 >^t$

$f_i = < F_{wi}\ M_{xi}\ M_{yi} >^t$ = externally applied nodal forces

$\mathbf{K}$ = stiffness matrix of the plate

$\mathbf{H}$ = stiffness matrix of the subbase

$\mathbf{d} = < d_1\ d_2\ d_3\ d_4 >^t$

$d_i = < w_i\ \theta_{xi}\ \theta_{yi} >^t$ = nodal displacements

in which superscript, t, indicates the transpose matrix.

In our study, the subbase is assumed to be a liquid foundation or a "Winkler foundation"; i.e., the vertical force at a nodal point in the subbase is proportional to the deflection at the node with a constant proportionality, the modulus of the subbase reaction, k. This assumption is employed in the analysis of concrete pavements (2,3,4).

Under this assumption, the stiffness matrix, $\mathbf{H}$, is banded and its components depend on the modulus of the subbase reaction, k.

2-2. Joint element

If the joints between blocks are narrow and filled with sand, frictional resistance will be developed in the joints and transmit part of the load to adjacent blocks (5). Since this mechanism is not clearly known, it is very difficult to treat the joint behavior theoretically. In our study, joints are assumed to be a set of springs which transmit part of the load proportional to the difference of the displacements between both sides of a joint.

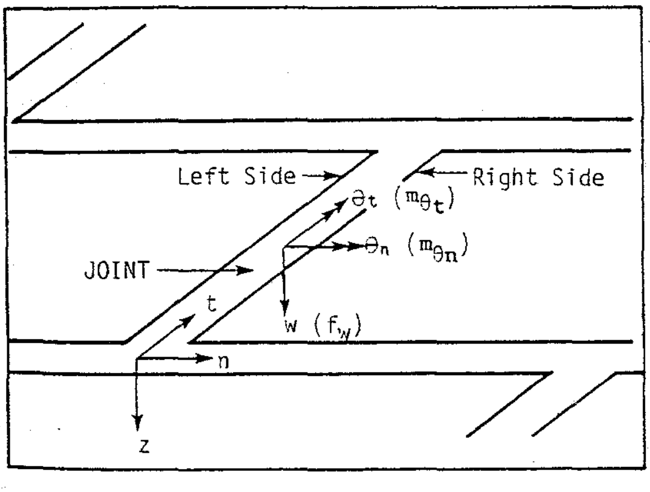


Figure 2: Coordinates and displacements of a joint.

Defining coordinates, forces and corresponding displacements of a joint as shown in Figure 2. this is represented by the following equation:

$$(\mathbf{f}_l - \mathbf{f}_r) = \mathbf{T} \cdot (\mathbf{d}_l - \mathbf{d}_r) \tag{2}$$

in which

$$\mathbf{f} = < f_w\ m_n\ m_t >^t$$
$$\mathbf{d} = < w\ \theta_n\ \theta_t >^t$$
$$\mathbf{T} = \begin{bmatrix} K_w & 0 & 0 \\ 0 & K_{\theta n} & 0 \\ 0 & 0 & K_{\theta t} \end{bmatrix} \tag{3}$$

Subscripts, r and l, indicate right and left side of the joint, respectively.

The quantities, K_w, $K_{\theta n}$ and $K_{\theta t}$, are the spring coefficients and represent the rigidity of the joint: shear, bending around n-axis and bending around t-axis, respectively. Their functions are diagrammatically illustrated in Figure 3.

This is not the strict physical model but a kind of reological one. Thus the values of the spring coefficients should be determined by experiments or field tests. Under this assumption, the formulation of the stiffness matrix of the joint element will be presented in the following.

We consider a joint element with four nodal points, 1, 2, 3 and 4, as shown in Figure 4. The deflection and rotations along with left and right sides of the joint element are assumed to be expressed by the following equations:

$$\begin{aligned}
w_l &= \alpha_1 + \alpha_2 t + \alpha_3 t^2 + \alpha_4 t^3 \\
\theta_{nl} &= -\alpha_2 - 2\alpha_3 t - 3\alpha_4 t^2 \\
\theta_{tl} &= + \alpha_5 + 2\alpha_6 t \\
w_r &= \beta_1 + \beta_2 t + \beta_3 t^2 + \beta_4 t^3 \\
\theta_{nr} &= -\beta_2 - 2\beta_3 t - 3\beta_4 t^2 \\
\theta_{tr} &= + \beta_5 + 2\beta_6 t
\end{aligned} \tag{4}$$

or

$$\mathbf{d}_l = \mathbf{B} \cdot \{\alpha\}, \quad \mathbf{d}_r = \mathbf{B} \cdot \{\beta\} \tag{5}$$

in which

$$\{\alpha\} = \{\alpha_1, \cdots\cdots, \alpha_6\}$$
$$\{\beta\} = \{\beta_1, \cdots\cdots, \beta_6\}$$
$$\mathbf{B} = \begin{bmatrix} 1 & t & t^2 & t^3 & 0 & 0 \\ 0 & -1 & -2t & -3t^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2t \end{bmatrix} \tag{6}$$

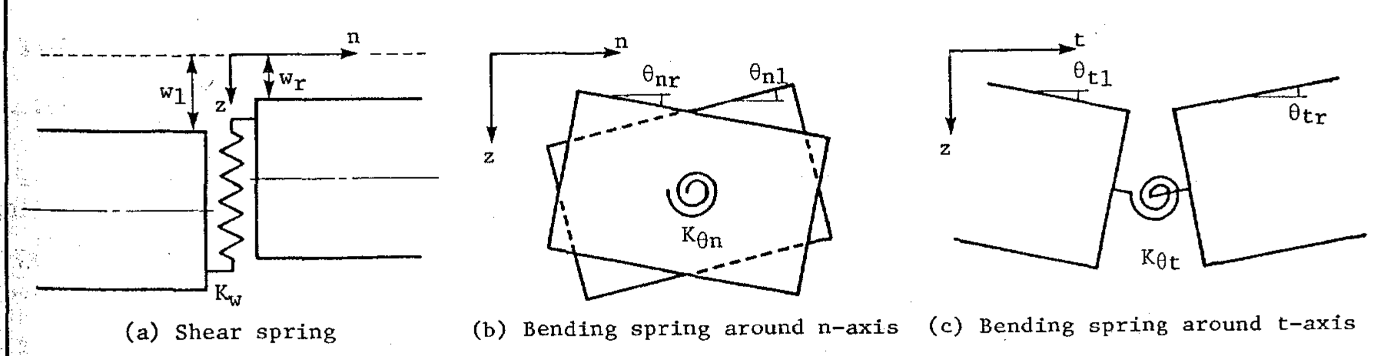


Figure 3: Modeling a joint as set of springs.

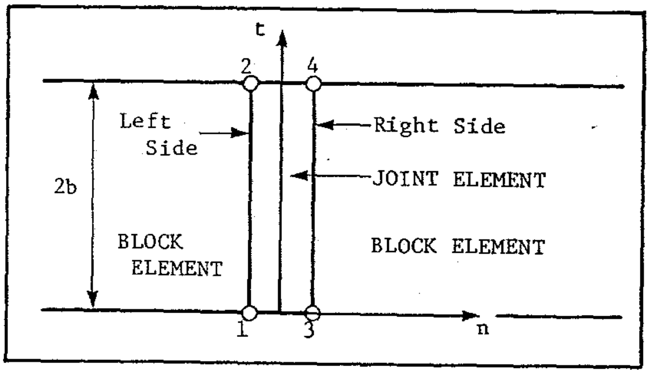


Figure 4: Joint element.

Using Eq. (5), the displacement vectors of four nodal points of the joint element can be related to the vectors, {α} and {β}, as follows:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{(t=0)} & \mathbf{0} \\ \mathbf{B}_{(t=2b)} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{(t=0)} \\ \mathbf{0} & \mathbf{B}_{(t=2b)} \end{bmatrix} \cdot \begin{bmatrix} \{\alpha\} \\ \{\beta\} \end{bmatrix} \tag{7}$$

or

$$\mathbf{d} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \cdot \begin{bmatrix} \{\alpha\} \\ \{\beta\} \end{bmatrix} \tag{8}$$

in which

$$\mathbf{C} = \begin{bmatrix} \mathbf{B}_{(t=0)} \\ \mathbf{B}_{(t=2b)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2b & 4b^2 & 8b^3 & 0 & 0 \\ 0 & -1 & -4b & -12b^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4b \end{bmatrix} \tag{9}$$

Solving Eq.(8) for {α} and {β}, the following expression is obtained:

$$\begin{bmatrix} \{\alpha\} \\ \{\beta\} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^{-1} \end{bmatrix} \cdot \mathbf{d} \tag{10}$$

in which

$$\mathbf{C}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -\frac{3}{4b^2} & -\frac{1}{b} & 0 & \frac{3}{4b^2} & \frac{1}{2b} & 0 \\ -\frac{1}{4b^3} & -\frac{1}{4b^2} & 0 & -\frac{1}{4b^3} & -\frac{1}{4b^2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{4b} & 0 & 0 & \frac{1}{4b} \end{bmatrix} \tag{11}$$

Substituting Eq.(10) into Eq.(5), the displacement vectors along with both sides of the joint element can be expressed by the nodal displacement vectors as follows:

$$\begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_r \end{bmatrix} = \begin{bmatrix} \mathbf{B}\cdot\mathbf{C}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}\cdot\mathbf{C}^{-1} \end{bmatrix} \cdot \mathbf{d} \tag{12}$$

Assuming that external forces are applied only at the nodal points and applying the virtual work principle to the joint element, the following equation is given:

$$\delta\mathbf{d}^t\cdot\mathbf{f} = \int_0^{2b} \delta(\mathbf{d}_1^t - \mathbf{d}_r^t)\cdot(\mathbf{f}_1 - \mathbf{f}_r)\,dt \tag{13}$$

Substitution of Eq.(13) by Eqs. (2) and (12) gives

$$\delta\mathbf{d}^t\cdot\mathbf{f} = \delta\mathbf{d}^t \cdot \begin{bmatrix} \mathbf{K}^* & -\mathbf{K}^* \\ -\mathbf{K}^* & \mathbf{K}^* \end{bmatrix} \cdot \mathbf{d} \tag{14}$$

in which

$$\mathbf{K}^* = \mathbf{C}^{-1t}\int_0^{2b} \mathbf{B}^t\cdot\mathbf{T}\cdot\mathbf{B}\,dt\,\mathbf{C}^{-1} \tag{15}$$

Since $\delta\mathbf{d}^t$ represents arbitrary virtual displacement, the following relation can be obtained:

$$\mathbf{f} = \begin{bmatrix} \mathbf{K}^* & -\mathbf{K}^* \\ -\mathbf{K}^* & \mathbf{K}^* \end{bmatrix} \cdot \mathbf{d}$$

or

$$\mathbf{f} = \mathbf{K}_j\cdot\mathbf{d} \tag{16}$$

in which

$$\mathbf{K}_j = \begin{bmatrix} \mathbf{K}^* & -\mathbf{K}^* \\ -\mathbf{K}^* & \mathbf{K}^* \end{bmatrix} \tag{17}$$

The matrix $\mathbf{K}_j$ represents the stiffness matrix of the joint element.

2-3 Idealization of interlocking block pavements

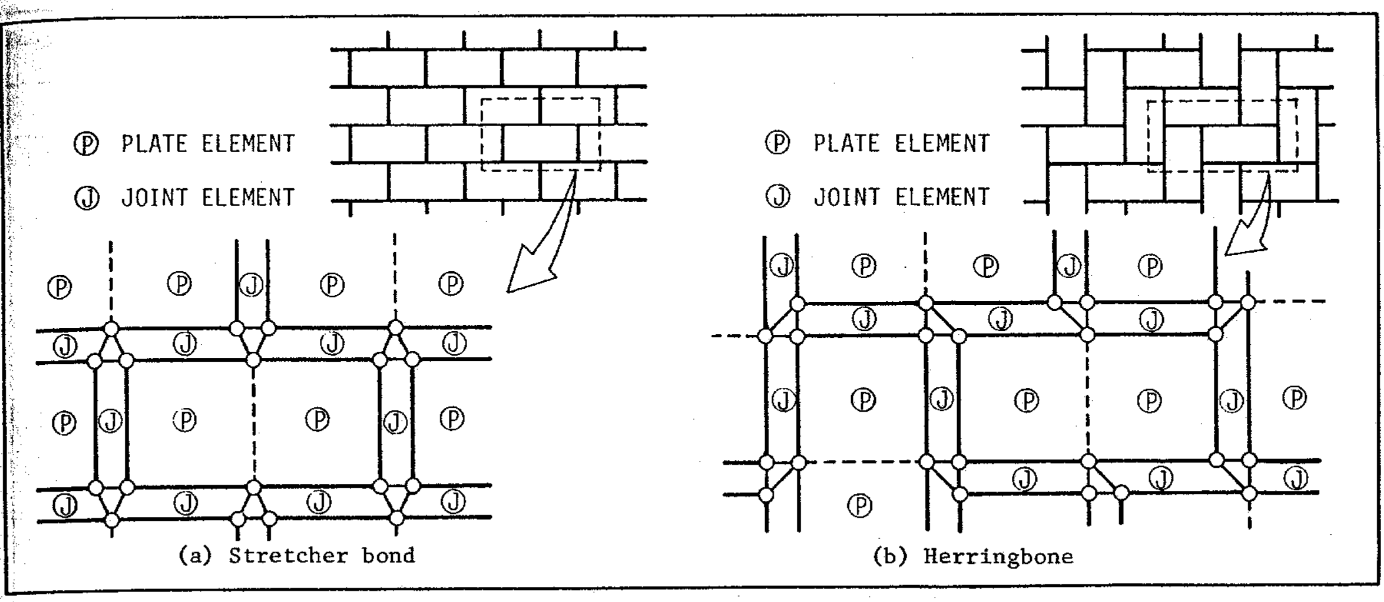
A interlocking block pavement is divided to plate elements and joint elements: the concrete blocks are divided to rectangular plate elments and joints are replaced by the joint elements which are presented in the previous section (Figure 5). Thus, by this method various laying patterns can be considered. Furthermore, if triangle plate elements are employed, more complex laying patterns can be handled. In this paper, two popular types of the laying pattern, stretcher bond and herringbone, will be discussed in the following sections (Figure 5).

3. CALCULATION AND CONSIDERATION

3-1. Spring coefficients

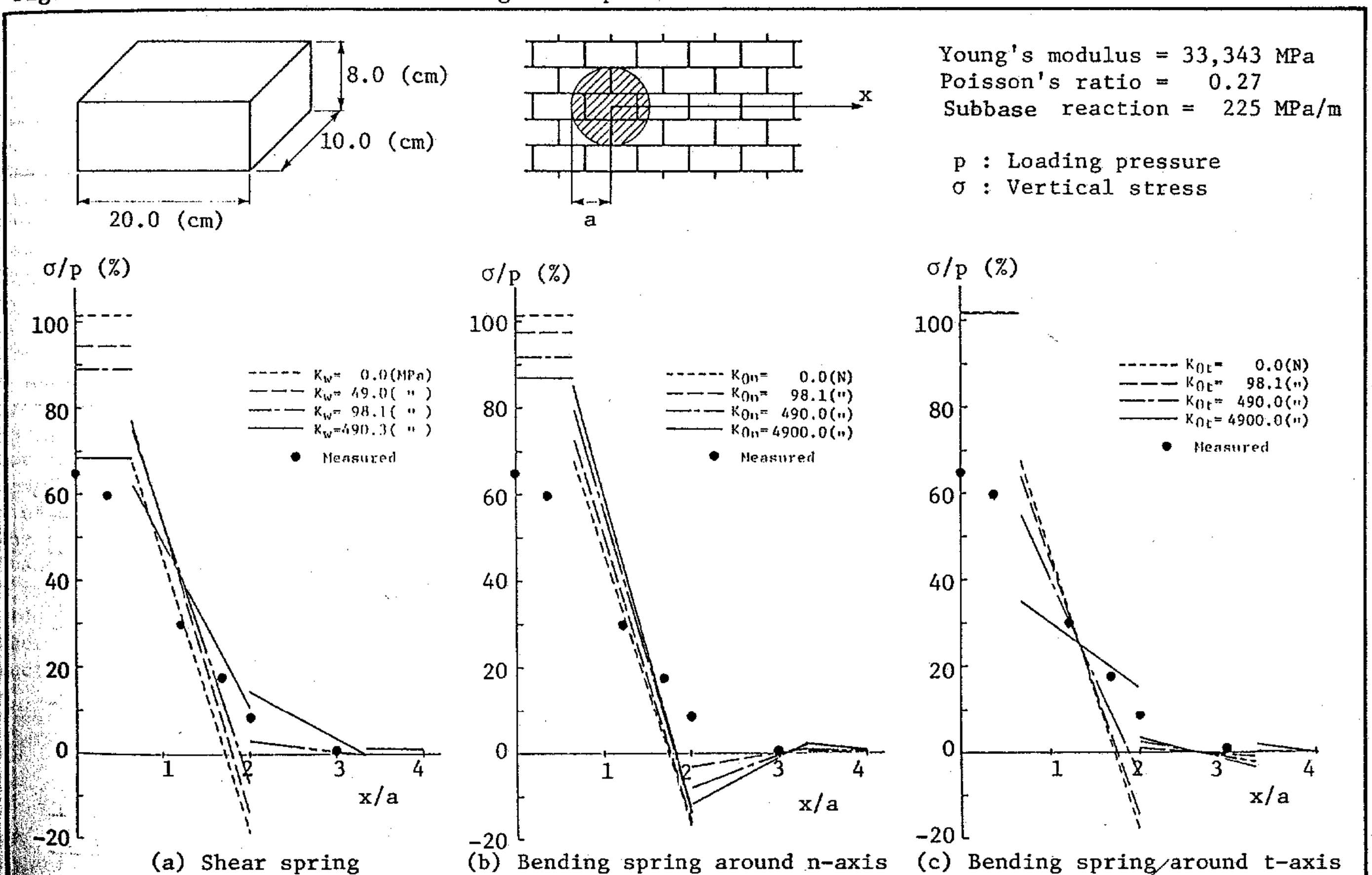
As previously described, the values of the spring coefficients should be determined by comparing computed results with experimental data. Unfortunately, since there is little available data (6), exact values of the spring coefficients could not be determined in this paper. Nevertheless, the function of each spring can be discussed by numerical calculations and tentative rough values of the spring coefficients can be obtained from a few experimental data.

Figure 6 shows the results computed by the computer program, changing the values of each spring coefficient under the same conditions. In this figure, the ordinate and abscissa are the ratio of the vertical stress at the top of the subbase to the loading pressure and the distance in x-direction normalized by the radius of the loading plate, respectively. The measured data from reference 6 are also plotted.



(a) Stretcher bond (b) Herringbone

Figure 5: Idealization of interlocking block pavements.



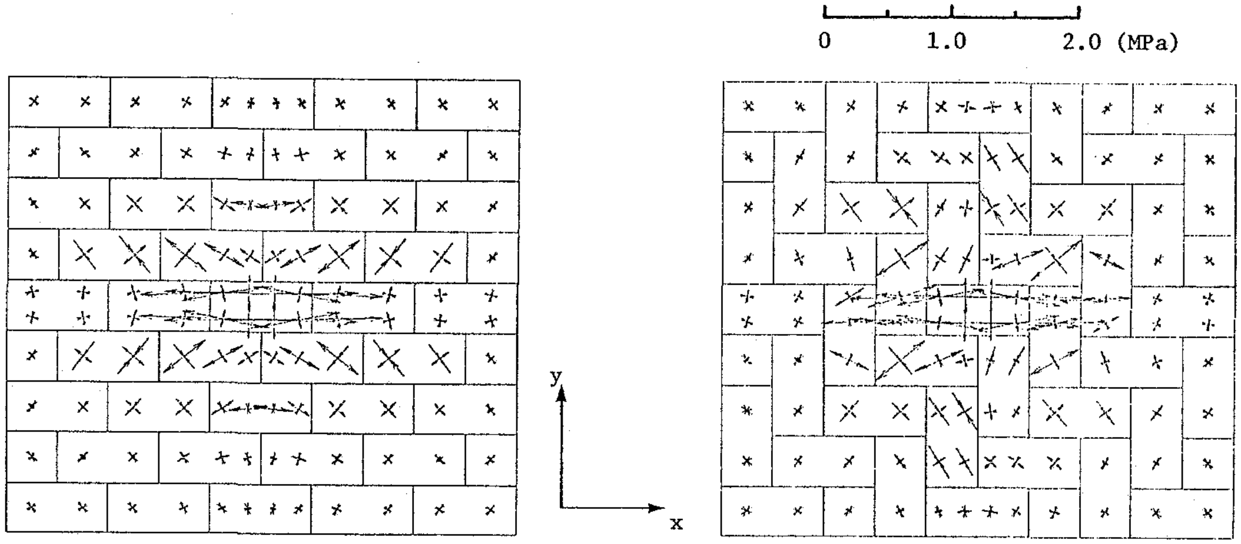
(a) Shear spring K_w (b) Bending spring around n-axis $K_{\theta n}$ (c) Bending spring around t-axis $K_{\theta t}$

Figure 6: Distribution of vertical stress at the top of the subbase.

Since the vertical stresses at the top of the subbase are smaller in Figure 6a than in Figures 6b and 6c, it can be said that the shear spring largely contribute to the load transmission. By comparing the computed results with the measured data, we assumed in the following calculations that joints only act as a shear spring: K_w = 490 MPa and $K_{\theta n}$ = $K_{\theta t}$ = 0 N. However, these values are expected to vary with supporting conditions and types of sand which fills joints. Thus more calculations and experiments are required to establish practical values of the spring coefficients.

3-2. Laying pattern

The method mentioned above can treat with various types of laying pattern of rectangular concrete blocks. We shall consider two types of



(a) Stretcher bond (b) Herringbone

Figure 7: Distribution of principle stresses.

Table 1: Maximum stresses and deflections varying with the subbase reaction,k.

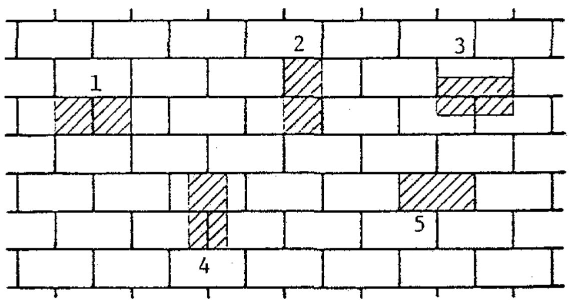
Subbase reaction,k (MPa/m)	Stretcher bond Maximum stress (MPa)	Stretcher bond Maximum deflection (cm)	Herringbone Maximum stress (MPa)	Herringbone Maximum deflection (cm)
49.0	2.71	0.107	3.38	0.068
98.1	2.55	0.058	2.69	0.042
245.2	2.13	0.027	2.24	0.023
490.3	1.95	0.016	2.00	0.014

laying pattern: a stretcher bond type and a herring bone type which are very popular and widely used.

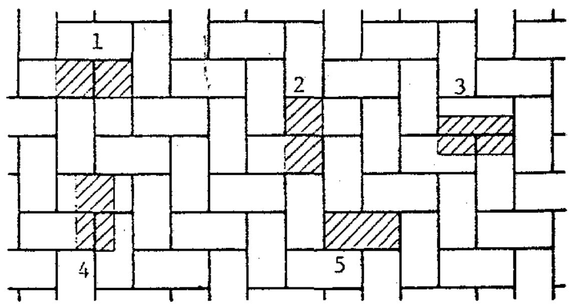
Figure 7 shows the difference of the stress distribution between these types of laying pattern under the same conditions. In the calculations, it is assumed that the load is applied on the center block and its pressure is 0.785MPa ,and that the modulus of the subbase reaction, k, is 245MPa/m. From this figure, it is clear that the stretcher bond type behaves anisotropically: the load transmission is larger in x-direction (the long edge direction). On the other hand, the herringbone type behaves almost isotropically. Table 1 shows the computed values of the maximum stresses and deflections of each type changing the modulus of the subbase reaction, k. It should be noted that, when k is small, the deflections of the stretcher type are larger than the herringbone type but the stresses of the stretcer bond is smaller than the herringbone type, and that, when k is relatively large, there is not much difference in these values between these two types of laying pattern.

3-3. Loading location

Even under the same load and laying pattern, the stresses and deflections may vary depending on whether or not the loading area crosses joints. In order to see the effect of the loading loca-



(a) Stretcher bond



(b) Herringbone

Figure 8: Loading locations.

Table 2: Maximum stresses and deflections varying with the loading location.

	Stretcher bond		Herringbone	
Loading location number (Figure 8)	Maximum stress (MPa)	Maximum deflection (cm)	Maximum stress (MPa)	Maximum deflection (cm)
1	2.34	0.078	3.03	0.064
2	2.48	0.070	3.75	0.064
3	2.83	0.075	3.44	0.065
4	4.22	0.067	5.07	0.061
5	2.73	0.075	3.10	0.064

tion. we computed the maximum stresses and deflection changing the loading location as shown in Figure 8. In the calculation, it is assumed that the tire imprint is 20x10 cm; the contact pressure is 9.81MPa and the modulus of the subbase reaction, k, is 245MPa/m.

Table 2 shows the computed results. In each case, there is little difference of the deflection, but the significant effect of the loading location on the stresses in the blocks is recognized. Particularly, at the loading location No. 4, the largest stress occurs in the block under the load.

4. CONCLUSIONS

We developed the computer program to analyze the mechanical behavior of the interlocking pavements and computed the stresses and deflections changing the laying pattern and loading condition. In the program, concrete blocks are divided to the rectangular plate elements and joints are replaced by the joint elements which are modeled as a set of springs.

The values of the spring coefficients of the joint element should be determined by comparing the computed results with experimental data. Owing to lack of available experimental data, we could not determine the exact values. Nevertheless, from computations and comparisons of computed results with a few experimental data, it was known that the shear spring strongly contributes to the load transmission. Then, the tentative values of the spring coefficients were determined. However, in order to establish their practical values, further calculations and experiments should be conducted.

To see the effect of the laying pattern, we considered two types: a stretcher bond type and a herringbone type. From the computed results, it was known that the stretcher bond type is of anisotropic behavior but the herringbone type is not, and that the maximum stresses and deflections hardly differ between these two types on relatively rigid subbase.

The loading location is expected to affect the stresses and deflections in the blocks because of the existence of the joints. From the calculations changing the loading location, it was proved that the loading location significantly affects the stresses in the block under the load.

5. REFERENCES

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